RAMANUJAN'S NOTEBOOKS (C. 1903–1910) · ACCESSIBLE TAGGED PDF/UA-1 EDITION

Ramanujan's Notebooks (c. 1903–1910) — Handwritten Master Theorem Manuscript

Accessibility Modality Note: This Tagged PDF/UA-1 file provides high-resolution vector equations paired with step-by-step spoken Alt descriptions for print and LMS archival use. For interactive symbol-by-symbol screen-reader navigation (NVDA + MathCAT, JAWS, VoiceOver), open the companion deliverable_accessible.html or deliverable_accessible.epub file included in this package.

Source Lecture Page 1

Ramanujan's Notebooks (c. 1903–1910) — Entry 11: Ramanujan's Master Theorem

Editorial Notation Note: In this original 1903–1910 Public Domain manuscript page, Srinivasa Ramanujan writes the vintage British factorial symbol (an L-shaped corner bracket around n) to denote n! (equivalently Gamma(n+1)) and uses '&c' for ellipsis (...). Below, modern standard MathML3 notation is used for full screen-reader clarity.

$$\int_0^\infty e^{-x} x^n \, dx = n!$$

[Page 1 · Eq #1] the integral from 0 to infinity, of e raised to the negative x power x to the n-th power, d x, equals n factorial

and hence (Ramanujan's Master Theorem):

$$\int_0^\infty x^{n-1} \left\{ \phi(0) - \frac{x}{1!}\phi(1) + \frac{x^2}{2!}\phi(2) - \cdots \right\} dx = (n-1)!\,\phi(-n)$$

[Page 1 · Eq #2] the integral from 0 to infinity, of x raised to the n minus 1 power times open brace phi of 0 minus x over 1 factorial phi of 1 plus x squared over 2 factorial phi of 2 minus dot dot dot close brace, d x, equals the quantity n minus 1, factorial phi of negative n

Solution sketch (term-by-term integration of power series):

$$\frac{f'(0)}{1!}\int_0^\infty e^{-rx}x^{n-1}\,dx = \frac{(n-1)!}{r^n}\cdot\frac{f'(0)}{1!}$$

[Page 1 · Eq #3] f prime of 0, over 1 factorial, times the integral from 0 to infinity, of e raised to the negative r x power x raised to the n minus 1 power, d x, equals the quantity n minus 1, factorial over r to the n-th power times f prime of 0, over 1 factorial

and so on. Adding up all the results we have:

$$\int_0^\infty x^{n-1}\left\{f(1)-\frac{x}{1!}f(r)+\frac{x^2}{2!}f(r^2)-\cdots\right\}dx=(n-1)!\,f\left(\frac{1}{r^n}\right)$$

[Page 1 · Eq #4] the integral from 0 to infinity, of x raised to the n minus 1 power times open brace f of 1 minus x over 1 factorial f of r plus x squared over 2 factorial f of r squared, minus dot dot dot close brace d x, equals the quantity n minus 1, factorial f of 1 over r to the n-th power

Let $f(r^n)=\phi(n)$, then $f(1/r^n)=\phi(-n)$.

$$\int_0^\infty x^{n-1}\left\{\phi(0)-x\phi(1)+x^2\phi(2)-\cdots\right\}dx=\frac{\pi\,\phi(-n)}{\sin(\pi n)}$$

[Page 1 · Eq #5] the integral from 0 to infinity, of x raised to the n minus 1 power times open brace phi of 0 minus x phi of 1 plus x squared phi of 2 minus dot dot dot close brace, d x, equals pi phi of negative n, over sine of pi n

$$\int_0^\infty x^{n-1}\left\{\phi(0)-\frac{x^2}{2!}\phi(2)+\frac{x^4}{4!}\phi(4)-\cdots\right\}dx=(n-1)!\,\phi(-n)\cos\left(\frac{\pi n}{2}\right)$$

[Page 1 · Eq #6] the integral from 0 to infinity, of x raised to the n minus 1 power times open brace phi of 0 minus x squared over 2 factorial phi of 2 plus x to the fourth power over 4 factorial phi of 4 minus dot dot dot close brace, d x, equals the quantity n minus 1, factorial phi of negative n cosine of the quantity pi n over 2

$$\int_0^\infty\left\{\phi(0)-\frac{x}{1!}\phi(1)+\frac{x^2}{2!}\phi(2)-\cdots\right\}\cos(nx)\,dx=\phi(-1)-n^2\phi(-3)+n^4\phi(-5)-\cdots$$

[Page 1 · Eq #7] the integral from 0 to infinity, of open brace phi of 0 minus x over 1 factorial phi of 1 plus x squared over 2 factorial phi of 2 minus dot dot dot close brace times cosine of n x, d x, equals phi of negative 1 minus n squared phi of negative 3 plus n to the fourth power phi of negative 5 minus dot dot dot

$$\int_0^\infty\left\{\phi(0)-x^2\phi(2)+x^4\phi(4)-\cdots\right\}\cos(nx)\,dx=\frac{\pi}{2}\left\{\phi(-1)-\frac{n}{1!}\phi(-2)+\frac{n^2}{2!}\phi(-3)-\frac{n^3}{3!}\phi(-4)+\cdots\right\}$$

[Page 1 · Eq #8] the integral from 0 to infinity, of open brace phi of 0 minus x squared phi of 2 plus x to the fourth power phi of 4 minus dot dot dot close brace times cosine of n x, d x, equals pi over 2 times open brace phi of negative 1 minus n over 1 factorial phi of negative 2 plus n squared over 2 factorial phi of negative 3 minus n cubed over 3 factorial phi of negative 4 plus dot dot dot close brace

---

Source & License Attribution: Source scan: Srinivasa Ramanujan's handwritten Master Theorem notebook page (c. 1903–1910, author died 1920), Public Domain (PD-old-100) via Wikimedia Commons. Transcribed into MathML3, spoken math audio, and PDF/UA-1 by EqScribe.